COMS30035, Machine learning: Combining Models 3, Trees

Edwin Simpson
edwin.simpson@bristol.ac.uk

Department of Computer Science, SCEEM
University of Bristol

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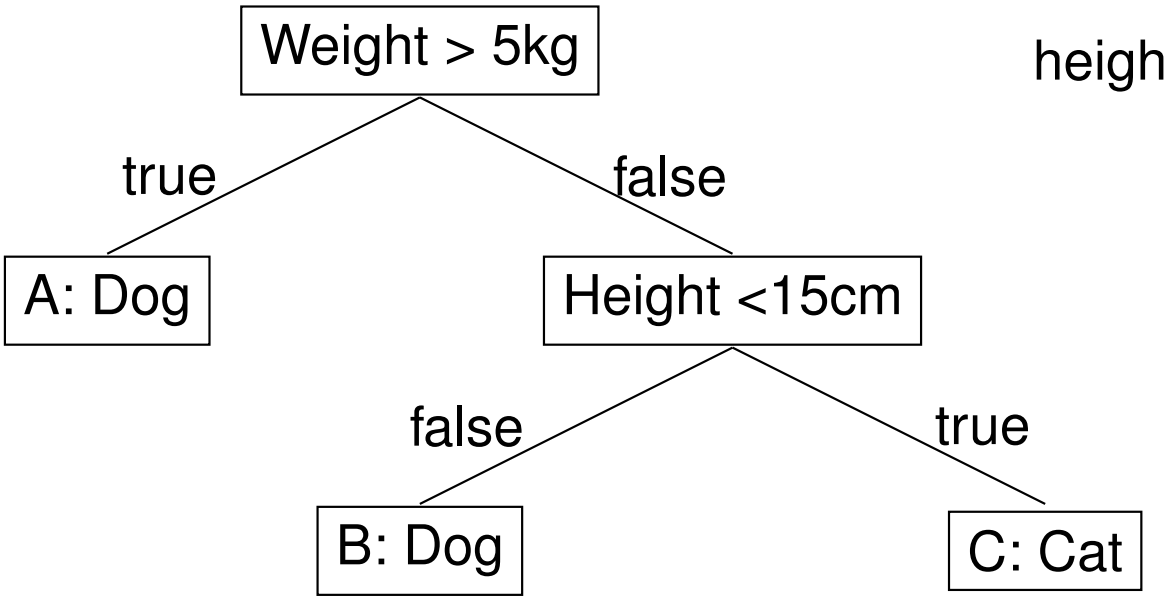
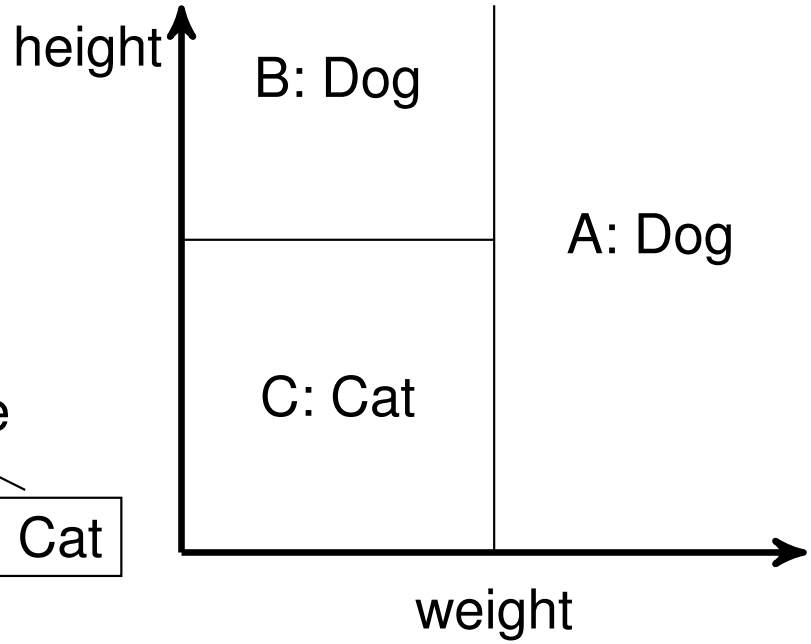
Agenda

- Model Selection
- Model Averaging
- Ensembles: Bagging
- Ensembles: Boosting and Stacking
- Tree-based Models
- Conditional Mixture Models
- Ensembles of Humans

Decision Trees





Decision Trees as Partitioning Input Space

- One model is responsible for assigning a decision for each region of input space;
- The correct model for an input $\boldsymbol{x}$ is chosen by traversing the binary decision tree, following the path from the top to a leaf.
- Leaf node is responsible for assigning a decision, such as a:
  - Class label;
  - Probability distribution over class labels;
  - Scalar value (for regression tasks).

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- Classification and Regression Trees (CART): one of many possible learning algorithms
- Objective: greedily minimise the error
  - Regression: sum-of-squares
  - Classification: cross-entropy as used in neural networks or Gini impurity

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  6. Prune back the tree to remove branches that do not reduce error by more than a small tolerance value, ϵ.

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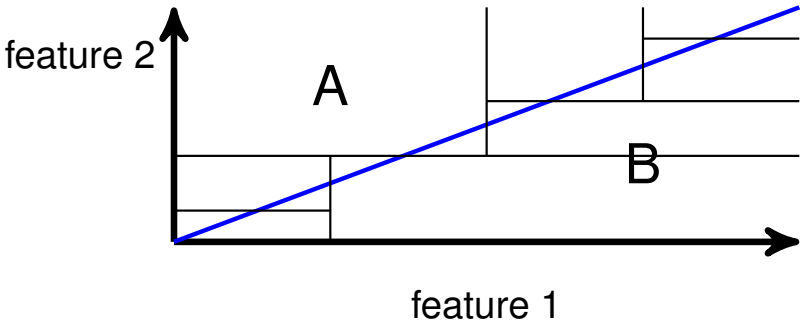
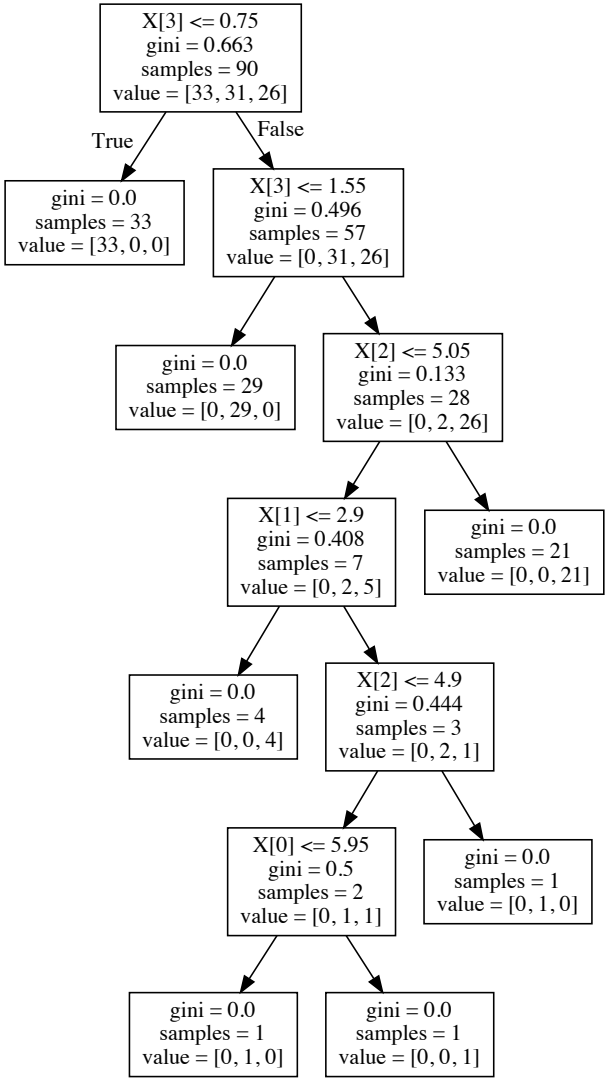
- If $C(T) \leq C(T_0)$ keep the pruned tree, else reinstate the pruned node.

Interpretability

- The sequence of decisions is often easier to interpret than other methods (think of neural networks);
- However, sometimes small changes to the dataset cause big changes to the tree;
- If the optimal decision boundary is not aligned with the axes of an input variable, we need a lot of splits.





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- As with bagging, combine predictions by taking mean/majority vote.
- Extremely effective in many applications (see Murphy (2012), Machine Learning: A Probabilistic Perspective, Section 16.2.5)

Now do the quiz!

Please do the quiz for this lecture on Blackboard.